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ANALYSIS OF THE CORRECTNESS OF A TWO-TEMPERATURE COMPUTATION METHOD

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Two methods of determining the heat transfer coefficient between components are compared on the basis of an exact solution of a model problem.

A multitemperature method [1-4] whose general principles are elucidated in [1] is used extensively at this time to model heat transport processes in heterogeneous media (granular, laminar, fibrous). This approach is based on taking the average of the thermophysical parameters with respect to each component in a macrovolume element, which results in a system of interrelated heat conduction equations. The connection between the heat flux between the components and their mean temperatures for which the Henry law is utilized [1]

$$q_{ij}^{*} = \alpha(\overline{T}_i - \overline{T}_j) \tag{1}$$

must be established to close the system.

Two methods are known for determining α: the "correlation" [1] and the linear radial heat flux methods [4, 5]. The problem of analyzing the correctness of the methods to determine the heat transfer coefficient between components is posed in this paper.

Let us examine a model heat propagation problem in a bilaminar composite of regular structure under boundary conditions of the second kind. The representative section of the material is displayed in Fig. 1. The thermophysical characteristics of the material components are considered independent of the temperature. Then we can write for an isolated section element

$$\lambda_{zi} T_{i,zz} + \lambda_{xi} T_{i,xx} = c_i T_{i,t}, \quad i = 1,\ 2, \tag{2}$$

$$T_i(0,\ z,\ x) = 0, \tag{2a}$$

$$-\lambda_{zi} T_{i,z}|_{z=0} = q_0(t), \quad \lambda_{zi} T_{i,z}|_{z=\text{n}} = q\text{n}(t), \tag{2b}$$

$$T_{i,x} = 0, \quad x = l_i, \tag{2c}$$

$$-\lambda_{x1} T_{1,x} = \lambda_{x2} T_{2,x}, \quad T_1 = T_2, \quad x = 0. \tag{2d}$$

1. Two-Temperature Theory. Let us introduce the concept of the mean temperature over a section $\overline{T}_i = \frac{1}{l_i}\int\limits_0^{l_i} T dx$. Then (2) can be converted into

$$\lambda_{zi} T_{i,zz} - c_i \overline{T}_{i,t} = \frac{(-1)^{i+1}}{l_i} q^{*}, \quad i = 1,\ 2,$$

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where $q^* = q_{1x}|_{x=0} = -\lambda_{x1}T_{1,x}|_{x=0}$.

To close the system of heat conduction equations obtained, it is necessary to find the dependence $q^* = f(\overline{T}_1, \overline{T}_2)$. The coefficient of heat transfer between components is determined in the "correlation" method on the basis of stochastic heat conduction equations of a micro-inhomogeneous medium, and

$$\alpha_k = 2\sqrt{3}\,\frac{l_1 l_2 \lambda_1 \lambda_2}{l_k^2 (l_1\lambda_1 + l_2\lambda_2)}, \tag{3}$$

is obtained in [1] for a bilaminar isotropic material, where ℓ_k is of the order of the characteristic dimension of the micro-inhomogeneities.

The idea of a "linear approximation" is proposed in [5]. It consists of assuming a linear dependence of the radial heat flux density on x, i.e., $q_{ix} = A_i(z)x$. Then by using (2c) and (2d) and the assumption made we obtain

$$\alpha_i = 3\lambda_1\lambda_2/(l_1\lambda_2 + l_2\lambda_1) \tag{4}$$

for the bilaminar isotropic material [5]. The equation (4) is refined in [4] for anisotropic components

$$\alpha_a = 3\lambda_{x1}\lambda_{x2}/(l_1\lambda_{x2} + l_2\lambda_{x1}). \tag{4a}$$

By using the "linear" approach, the heat transfer coefficient between two coaxial cylinders under ideal thermal contact can be determined

$$\alpha_\ell = \frac{12\lambda_{x1}\lambda_{x2}(R_2 + R_1)}{3\lambda_{x2}(R_2 + R_1)R_1 + \lambda_{x1}(R_2 - R_1)(5R_2 + 3R_1)}. \tag{4b}$$

Taking account of the thermal contact resistance between the components results in the expression

$$\alpha_R = \alpha(1 + R_{con}\alpha)^{-1}. \tag{4c}$$

Let us solve the system of equations (2) by the two-temperature method for the α determined by (3) or (4). Applying the Laplace transform in time and the Fourier cosine transform in z we have

$$\overline{T}_{1L} = Q_L(\alpha_l + \lambda_{z2}\psi + c_2 p)\,\Gamma^{-1},$$
$$\overline{T}_{2L} = Q_L(\alpha_l + \lambda_{z1}\psi + c_1 p)\,\Gamma^{-1},$$

where

$$\Gamma = c_1c_2[(p + j)^2 - \beta^2], \quad j = \frac{\psi}{2}(a_{z1} + a_{z2}) + \alpha_0,$$

$$\beta^2 = \left[\psi\,\frac{a_{z1} - a_{z2}}{2} + \frac{\alpha}{2}\,\frac{l_2c_2 - l_1c_1}{l_1l_2c_1c_2}\right]^2 + \alpha(l_1l_2c_1c_2)^{-1},$$

$$\alpha_0 = \frac{\alpha}{2}(l_2c_2 + l_1c_1)(l_1l_2c_1c_2)^{-1}, \quad \psi = [n\pi/H]^2,$$

$$\alpha_l = \alpha(l_1 + l_2)/(l_1l_2), \quad a_{zi} = \lambda_{zi}/c_i, \quad i = 1,\ 2,$$

$$Q_L = \int_0^\Phi \exp(-pt)\,Q\,dt, \quad Q = q_0 - (-1)^n q_n.$$

To obtain the originals $\overline{T}_1$, $\overline{T}_2$, the inverse Laplace and Fourier transforms must be performed for the transforms $\overline{T}_{iL}$, i = 1, 2. Let us consider that q_0 = const, $q_n = 0$, $\lambda_{z1} > \lambda_{z2}$; then we have

$$\overline{T}_i = \frac{q_0}{Hc_i}\left[\frac{B_i t}{\alpha_0} + U\left(1 - \frac{B_i}{\alpha_0}\right) + \sum_{n=1}^{\infty}\left\{\frac{2A_i}{vw} - \frac{\exp(-vt)}{\beta}\left(\frac{A_i}{v} - 1\right) - \exp[-wt]\left(\frac{A_i}{w} - 1\right)\frac{1}{\beta}\right\}\cos\left(\frac{n\pi z}{H}\right)\right], \quad i = 1,\ 2, \tag{5}$$

where

$$B_1 = \frac{\alpha}{2}\,\frac{l_2c_2 + l_1c_2}{l_1l_2c_2^2}, \quad B_2 = \frac{\alpha}{2}\,\frac{c_1l_1 + c_1l_2}{l_1l_2c_1^2}, \quad v = \gamma - \beta,$$

$$w = \gamma + \beta, \quad U = (1 - \exp(-2\alpha_0 t))/(2\alpha_0), \quad A_i = 2B_i + a_{zi}\psi.$$

2. Exact Solution. Applying the Laplace and Fourier transforms to the system (2) we obtain

$$T_{i,xxL} - \frac{1}{a_{xi}}(p + a_{zi}\psi) T_{iL} = -Q_L/\lambda_{xi}, \quad i = 1, 2 \tag{6}$$

in transform space. The solution of the system (6), (2c) and (2d) can be represented in the form (i, j = 1, 2; i ≠ j)

$$T_{iL} = \frac{Q_L}{p\varphi_i\lambda_{xi}} + (-1)^{i+1} Q_L \Phi \operatorname{sh}(\sqrt{\varphi_j}\, l_j) \operatorname{ch}(\sqrt{\varphi_i}\,(l_i - x)) \sqrt{\varphi_j}\, \lambda_{xj}/\Pi_0,$$

where

$$\varphi_i = \frac{p + a_{zi}\psi}{a_{xi}}, \quad \Phi = \frac{\lambda_{x1}\varphi_1 - \lambda_{x2}\varphi_2}{\lambda_{x1}\lambda_{x2}\varphi_1\varphi_2},$$

$$\Pi_0 = \lambda_{x2}\sqrt{\varphi_2}\, \operatorname{ch}(\sqrt{\varphi_1}\, l_1) \operatorname{sh}(\sqrt{\varphi_2}\, l_2) + \lambda_{x1}\sqrt{\varphi_1}\, \operatorname{ch}(\sqrt{\varphi_2}\, l_2) \operatorname{sh}(\sqrt{\varphi_1}\, l_1).$$

Let us execute the inverse Laplace and Fourier transforms by noting that it is recommended to perform them for a specific kind of Q_L because formulas for the general case of Q_L can only be written as a convolution. Let us consider that q_0 = const, $q_n = 0$, $\lambda_{z1} > \lambda_{z2}$. We use the theorem of expansion of the transforms [6] to obtain the Laplace originals. The expression (7) is the ratio of generalized polynomials. To find the roots of the denominator we examine the equation $p\varphi_1\varphi_2\Pi_0 = 0$. Its analysis shows that there are three groups of simple roots:

1) Roots of the equations $\varphi_i = 0$, i = 1, 2;

2) Roots p_{nm} determinable from the equation

$$\lambda_{x2} g_2 \operatorname{ch}(g_1 l_1) \sin(g_2 l_2) - \lambda_{x1} g_1 \operatorname{sh}(g_1 l_1) \cos(g_2 l_2) = 0,$$

3) Roots p_{kn} determinable from the equation

$$\lambda_{x2} r_2 \cos(r_1 l_1) \sin(r_2 l_2) + \lambda_{x1} r_1 \cos(r_2 l_2) \sin(r_1 l_1) = 0,$$

where $r_i^2 = (p_{kn} - a_{zi}\psi)/a_{xi}$, $i = 1, 2$, $g_1^2 = (a_{z1}\psi - p)/a_{x1}$, $g_2^2 = (p - a_{z2}\psi)/a_{x2}$.

Then p = 0 is a double root. As a result of the inverse transformations in t and z we have (i, j = 1, 2; i ≠ j)

$$T_i = \frac{q_0}{H}\left[\frac{t}{c_s} - (-1)^i \Delta c \frac{c_j l_j}{c_l} U_c^i + (-1)^i \frac{\lambda_{xi}\Delta c}{\sqrt{a_{xj}}} \sum_{k=1}^{\infty} U_{k0}^i + \right.$$

$$+ 2\sum_{n=1}^{\infty}\left\{\frac{1}{\psi\lambda_{zi}} - (-1)^i \frac{\Delta\lambda}{\psi} U_0^i - \sum_{m=1}^{A_m} L_{nm} U_s^i \exp(-p_{nm}t) + \right. \tag{8}$$

$$\left.\left. + (-1)^i \sum_{k=1}^{\infty} L_{kn} U_{kn}^i \exp(-p_{kn}t)\right\} \cos\left(\frac{n\pi z}{H}\right)\right],$$

where

$$c_s = \frac{l_2 c_2 + l_1 c_1}{l_1 + l_2}, \quad \Delta c = \frac{c_1 - c_2}{c_1 c_2}, \quad c_l = c_s(l_1 + l_2),$$

$$U_c^i = \frac{l_j^2}{6a_{xj}} + \frac{(l_i - x)^2}{2a_{xi}} - \frac{1}{c_l}\left[l_2 c_2\left(\frac{l_1^2}{2a_{x1}} + \frac{l_2^2}{6a_{x2}}\right) + \right.$$

$$\left. + l_1 c_1\left(\frac{l_2^2}{2a_{x2}} + \frac{l_1^2}{6a_{x1}}\right)\right],$$

$$U_{k0}^i = \frac{\sin(r_{j0} l_j)\cos(r_{i0}(l_i - x))}{p_{k0}\sqrt{p_{k0}}\, \Pi_p(0, p_{k0})} \exp(-p_{k0}t), \quad \Pi_p(0, p_{k0}) = \left.\frac{\partial \Pi_0}{\partial p}\right|_{p=-p_{k0}},$$

$$U_0^i = \lambda_{xj}\sqrt{\varepsilon_j}\, \operatorname{sh}(\sqrt{\varepsilon_j\psi}\, l_j) \operatorname{ch}(\sqrt{\varepsilon_i\psi}\,(l_i - x))/\Pi_0(n, p = 0),$$

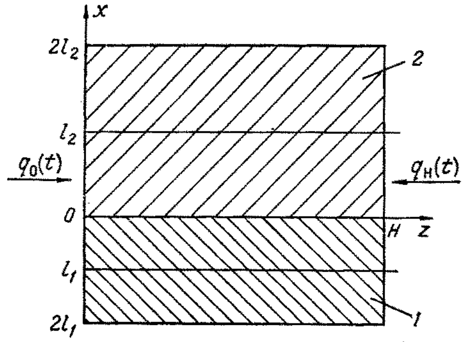

Fig. 1. Representative volume for a bilaminar composite of regular structure: 1) first layer; 2) second lyaer; H is the material thickness

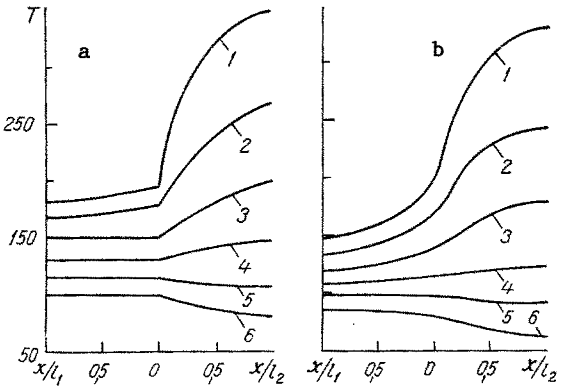

Fig. 2. Temperature distribution in material sections $\bar{z}$ = z/H: 1) $\bar{z}$ = 0; 2) 0.01; 3) 0.02; 4) 0.03; 5) 0.04; 6) 0.05 for a) Ke (0); b) Ke (0; λ_{z1} = 20 W/(m·K)) T, K.

$$U_{kn}^{i}=\frac{\lambda_{xj}r_j\sin(r_jl_j)\cos(r_i(l_i-x))}{p_{kn}(p_{kn}-a_{z1}\psi)(p_{kn}-a_{z2}\psi)\Pi_p(n,\ p_{kn})},\quad \varepsilon_i=\frac{a_{zi}}{a_{xi}},$$

$$\Pi_p(n,\ p_{kn})=\left.\frac{\partial\Pi_0}{\partial p}\right|_{p=-p_{kn}},\quad \Delta\lambda=(\lambda_{z1}-\lambda_{z2})/(\lambda_{z1}\lambda_{z2}),$$

$$U_s^1=\frac{\sin(g_2l_2)\operatorname{ch}(g_1(l_1-x))}{p_{nm}\lambda_{x1}g_1^2\,g_2\Pi_p(\beta)},\quad U_s^2=\frac{\operatorname{sh}(g_1l_1)\cos(g_2(l_2-x))}{p_{nm}\lambda_{x2}g_2^2\,g_1\Pi_p(\beta)},$$

$$\Pi_p(\beta)=\frac{1}{2}[\cos(g_2l_2)\operatorname{ch}(g_1l_1)(c_2l_2+c_1l_1)+\operatorname{sh}(g_1l_1)\sin(g_2l_2)\times$$

$$\times\left\{\frac{\lambda_{x1}g_1l_2}{a_{x2}g_2}-\frac{\lambda_{x2}g_2l_1}{a_{x1}g_1}\right\}+\operatorname{ch}(g_1l_1)\sin(g_2l_2)\frac{c_2}{g_2}+\frac{c_1}{g_1}\cos(g_2l_2)\times$$

$$=\operatorname{sh}(g_1l_1)],\quad L_{ij}=(\lambda_1-\lambda_2)\psi-(c_1-c_2)p_{ij}.$$

The number A_m determines the quantity of roots p_{nm} for each n.

To obtain the mean temperature $\langle T_i \rangle$ over a component section, (8) must be integrated within limits of the layer. We make numerical estimates for a material with the following thermophysical characteristics: $\lambda_{z1} = \lambda_{x1}$ = 160 W/(m·K); $\lambda_{z2} = \lambda_{x2}$ = 20 W/(m·K); $c_1 = c_2$ = $3.23 \cdot 10^6$ J /(m^3·K) and the geometric parameters ℓ_1 = 0.4 mm; ℓ_2 = 0.8 mm; H = 20 mm. Let Ke(0) denote the set of parameters listed. We let Ke(0; f = y) denote a composite material different from Ke(0) by the value of the parameter f = y. Let us give the thermal flux density on the boundary q_0 = 10^7 W/m^2.

Let us note that (8) is simplified for the selected thermophysical characteristics of the components:

1) p_{kn} (k = 1, 2,..., n = 0, 1, 2,...), $p_{kn} \geq 60$, consequently, terms containing exp $(-p_{kn}t)$ become negligible for t ~ 1 sec;

2) A_m = 1 for all n ≤ 200;

3) Δc = 0, i = 1, 2.

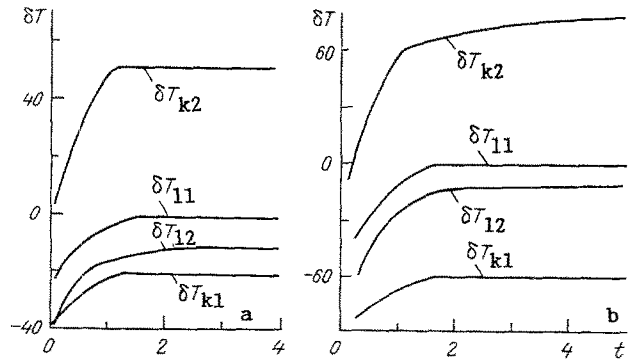

Fig. 3. Absolute error of two-temperature models for materials: a) Ke(0); b) Ke (0; ℓ_1 = 0.2 mm, ℓ_2 = 1.0 mm), δT, K; t, sec.

On the basis of computations N = max (n) was selected equal to 200 which assures an accuracy on the order of 2%. Let us note that N is a function of the material thermophysical and geometric characteristics.

The temperature distribution in different sections of a laminar material is displayed in Fig. 2 at the times t = 0.1 sec for Ke(0) and Ke (0; λ_{x1} = 20 W/(m·K)), respectively.

The mean temperatures of the components were computed by the two-temperature theory (5) for the material Ke(0): ($\bar{T}_{ki}$) by the "correlation" method (3) and ($\bar{T}_{li}$) by the linear approximation (4). Since the maximal difference between the component temperatures will be on the composite boundary [4] at the point z = 0, the approximate values were compared to the exact value at this point.

Let us introduce δT_{ci} and δT_{ki} characterizing the absolute error of the methods

$$\delta T_{li} = \langle T_i \rangle - \bar{T}_{li}, \quad \delta T_{ki} = \langle T_i \rangle - \bar{T}_{ki}, \quad i = 1, 2.$$

The results of comparing the methods for computing the temperature field of the material Ke(0) are represented in Fig. 3a, and for the material Ke (0; ℓ_1 = 0.2 mm, ℓ_2 = 1, 9 mm) in Fig. 3b.

Analysis of the computation results shows that the "linear approximation" method is more exact since it yields complete agreement with the exact solution for the component 1 for t > 1.0 sec. The absolute error is ~10° for the less heat-conductive component 2 and it depends weakly on the geometric dimensions of the component. The greatest error in the "linear approximation" occurs at the initial stages of material heating when the radial thermal flux density depends nonlinearly on the coordinate x. The reason for this is that only peripheral zones of the component take part in inter-component heat transfer in the initial heating stage. As all the components become involved in the inter-component heat transfer, a temperture profile is formed in its sections that is almost a linear approximation for q_{ix}, which corresponds to a parabolic temperture distribution over the section of a laminar composite.

Therefore, an exact solution is obtained for the temperture field of a bilaminar composite for constant heat flux at the boundary, on whose basis two methods of computing the coefficient of heat transfer between the components are compared. It is shown that the "linear" radial heat flux method is more exact as compared with the "correlation" method.

NOTATION

T, temperature; $\bar{T}$, temperature averaged over the section; δT, absolute error of the temperature; T_L, transform of the temperature; t, time; x, z, space coordinates; ℓ, H, R, geometric characteristics of the representative section; q, thermal flux density; q^*_{ij}, thermal flux density from the i component to the j component; c, λ, a) coefficients of volume specific heat, heat conductivity, and thermal diffusivity; α, heat transfer coefficient between the components; R_{con}, contact thermal resistance factor; p, Laplace transform parameter; n, Fourier cosine-transform parameter; ,xx = $\partial^2/\partial x^2$ comma before the subscripts denotes differentiation with respect to the appropriate variable.

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